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APPLICATION NOTE - 023

Harmonic analysis in power applications

By convention, waveforms are considered as starting (t = 0) at their peak values, ie:

$V(t) = \cos(\omega t)$
$A(t) = \cos(\omega t + \theta)$ where θ is the relative phase angle

So for harmonic analysis, using the complex notation:

$h_n = a_n + j\, b_n$

the in-phase and quadrature values of the n^{th} harmonic of a periodic waveform, $v(\phi)$, are given by:

$$a_n = \sqrt{2}/2\pi \int_{-\pi}^{\pi} v(\phi).\cos(n\phi)\, d\phi$$

$$b_n = \sqrt{2}/2\pi \int_{-\pi}^{\pi} v(\phi).\sin(n\phi)\, d\phi$$

For a square wave:

$$\begin{aligned} v(\phi) &= -A \quad \text{for} \quad -\pi < \phi < -\pi/2 \\ &= +A \quad \text{for} \quad -\pi/2 < \phi < \pi/2 \\ &= -A \quad \text{for} \quad \pi/2 < \phi < \pi \end{aligned}$$

Then

$$a_n = A\sqrt{2}/2\pi n \left(-[\sin(n\phi)]_{-\pi}^{-\pi/2} + [\sin(n\phi)]_{-\pi/2}^{\pi/2} - [\sin(n\phi)]_{\pi/2}^{\pi} \right)$$

$$= A\sqrt{2}/2\pi n\, (-\sin(-n\pi/2) + \sin(n\pi/2) - \sin(-n\pi/2) + \sin(n\pi/2))$$
$$= A\, 2\sqrt{2}/\pi n \sin(n\pi/2)$$

and

$$b_n = A\sqrt{2}/2\pi n \left(-[-\cos(n\phi)]_{-\pi}^{-\pi/2} + [-\cos(n\phi)]_{-\pi/2}^{\pi/2} - [-\cos(n\phi)]_{\pi/2}^{\pi} \right)$$

$$= A\sqrt{2}/2\pi n\, (-\cos(-n\pi) + \cos(n\pi))$$
$$= 0$$

So it can be seen that for

$n = 1, 5, 9 \ldots$	$a_n = A\, 2\sqrt{2}/\pi n$	(relative magnitude 1/n, phase 0°)
and for $n = 3, 7, 11 \ldots$	$a_n = -A\, 2\sqrt{2}/\pi n$	(relative magnitude 1/n, phase 180°)
and for even n	$a_n = 0$	
and in all cases	$b_n = 0$	